QUASI-P NORMAL AND QUASI-n-P NORMAL COMPOSITION, WEIGHTED COMPOSITION AND COMPOSITE MULTIPILICATION OPERATORS

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Abstract — In this paper we characterized quasi-P normal, quasi-n-P normal composition, weighted composition, composite multiplication operators

Keywords - quasi-P normal, quasi-n-P normal, composition operators

1 INTRODUCTION

An operator T is said to be Self adjoint operator, If T satisfies $T^* = T$. An operator T is said to be normal, If T satisfies $TT^* = T^*T$. An operator T is said to be n-power normal, If T satisfies $T^*T^n = T^nT^*$. An operator T is said to be binormal, If $TT^*$and T^*T commute (i.e) $(TT^*)T^*T = T^*T(TT^*)$. An operator T is said to be quasi normal, If T and T^*T commute. An operator T is said to be quasi-n-normal, If T and T^*T^ncommute. An operator T is said to be quasi-P normal, If $(T+T^*)$ and T^*Tcommute. An operator T is said to be quasi-n-P normal, If $(T+T^*)$ and T^*T^ncommute.

SOME PROPERTIES OF QUASI-P NORMAL AND QUASI–n-P NORMAL OPERATORS

THEOREM-1: If $T \in B(H)$ is isometry, Then T is Quasi-p normal.
PROOF: Let T is isometry, we have $T^*T = I$
Now

$$(T+T^*)(T^*T) = (T+T^*).I = (T+T^*) \quad (1)$$
$$(T^*T)(T+T^*) = I.(T+T^*) = (T+T^*) \quad (2)$$

From (1) and (2) are same.
Hence T is quasi-p normal.

THEOREM -2 : Every quasinormal operator is quasi-p normal.
PROOF: Let T is quasinormal operator, Then

$$T(T^*T) = (T^*T)T \quad (3)$$

Taking adjoint on the both side of (3) we get,

$$(T(T^*T))^* = ((T^*T)T)^*$$
$$T^*TT^* = T^*T^*T \quad (4)$$

$$(T+T^*)(TT^*) = TT^*T + T^*T^*T$$
$$= (T^*T)T + (T^*T)T^*$$
$$= (T^*T)(T+T^*) \quad (5)$$

From (5), Hence T is quasi-p normal.

THEOREM-3: If T is a quasi-n-p normal and μ is any scalar which is real. Then μT is also a quasi-n-p normal operator.
PROOF: Let T is quasi-n-p normal operator, Then

$$(T+T^*)(T^*T^n) = (T^*T^n)(T+T^*)$$

If μ is any scalar which is real, Then

$$(\mu T)^* = \bar{\mu}T^* = \mu T^*, \ \mu \text{ is real}$$
$$((\mu T + (\mu T)^*)((\mu T)^*(\mu T)^n) = (\mu T + \bar{\mu}T^*)((\bar{\mu}T^*)(\mu T)^n)$$
$$= \mu^{2+n}(T+T^*)(T^*T^n) \quad (6)$$
$$((\mu T)^*(\mu T)^n)(\mu T + (\mu T)^*) = ((\bar{\mu}T^*)(\mu T)^n)(\mu T + \bar{\mu}T^*)$$
$$= \mu^{2+n}(T^*T^n)(T+T^*) \quad (7)$$

From (6) and (7), We get,
Therefore μT is quasi-n-p normal operator.

THEOREM-4: If T is a self-adjoint operator then T is a quasi-n-p normal.
PROOF: Let T is a self-adjoint operator

$$T^* = T \quad (8)$$

Now,

$$(T+T^*)(T^*T^n) = (T+T)(TT^n)$$
$$= 2T^{2+n} \quad (9)$$
$$(T^*T^n)(T+T^*) = (TT^n)(T+T)$$
$$= 2T^{2+n} \quad (10)$$

From (9) and (10), Hence T is quasi-n-p normal.

THEOREM-5: Let T be a quasi-n-p normal operator on a Hilbert space H. Let S be a self-adjoint operator for which T and S commute, Then ST is also a quasi-n-p normal.

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THEOREM-6: Let T ∈ B(H) be a quasi-n-p normal operator which is unitary equivalent to S if and only if $TU = UT$ and $T^*U = UT^*$. Then S is a quasi-n-p normal.

THEOREM-7: If T is a quasi-n-normal operator which is n-power normal also, Then T is quasi-n-p normal operator.

THEOREM-8: Let T_1 and T_2 be a two quasi-n-p normal operators which each is the adjoint of the other, Then T_1T_2 is a quasi-n-p normal operator.

THEOREM-9: If T be a self adjoint operator on a Hilbert space H and S be any operator on H, Then S^*TS is a quasi-n-p normal operator on H

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THEOREM-10: If T is a quasi-n-p normal. Then T^* is a quasi-n-p normal operator.

PROOF: Let T is a quasi-n-p normal.

$$(T + T^*)(T^*T^n) = (T^*T^n)(T + T^*) \tag{11}$$

Substituting T^* for T in (11), We have

$$(T^* + T)(TT^{*n}) = (TT^{*n})(T^* + T) \tag{12}$$

Hence T^* is quasi-n-p normal.

THEOREM-11: Let T be a quasi-n-p normal operator. Which is a unitary operator also, then T^{-1} is a quasi-n-p normal.

THEOREM-12: Let T ∈ B (H), $A = (T^*T^n) + (T + T^*)$ and $B = (T^*T^n) - (T + T^*)$, Then T is quasi-n-p normal operator if and only if A commutes with B.

THEOREM-13: Let T ∈ B (H), $X = (T^*T^n)(T + T^*)$, $A = (T^*T^n) + (T + T^*)$ and $B = (T^*T^n) - (T + T^*)$, Then T is quasi-n-p normal operator if and only if X commutes with A and B.

QUASI-P NORMAL AND QUASI–n-P NORMAL COMPOSITION OPERATORS

Let C be the composition operator on $L^2(\mu)$. Then the adjoint C^* is given by $C^*f = h.E(f) \circ T^{-1}$ for f in $L^2(\mu)$.

Lemma-14: Let P be the projection of $L^2(X, \sum, \mu)$ onto $\overline{R(C)}$. Then

(i) $C^*Cf = hf$ and $CC^*f = (h \circ T)Pf$, for all $f \in L^2(\mu)$.

(ii) $\overline{R(C)} = \{f \in L^2(\mu) : f \text{ is } T^{-1}(\sum) \text{ measurable}\}$.

(iii) If f is $T^{-1}(\sum)$ measurable, g and fg belong to $L^2(\mu)$, then $P(fg) = fP(g)$,(f need not be in $L^2(\mu)$).

(iv) $(C^*C)^k f = h_k f$ for $k \in N$.

(v) $(CC^*)^k f = (h \circ T)_k P(f)$.

(vi) E is the identity operator on $L^2(\mu)$ if and only if $T^{-1}(\sum) = \sum$.

The following theorem characterizes the quasi-p normal and quasi – n -p normal composition operators.

THEOREM-15: Every quasinormal composition operator is quasi-p normal operator.

PROOF: Let C is quasinormal composition operator, then

$$C(C^*C) = (C^*C)C \tag{13}$$

Taking adjoint on both side in (13), We get

$$(C(C^*C))^* = ((C^*C)C)^*$$
$$C^*CC^* = C^*C^*C$$
$$\begin{aligned}(C + C^*)(C^*C) &= (CC^*C)(C^*C^*C)\\ &= C^*CC + C^*CC^*\\ &= (C^*C)(C + C^*)\end{aligned}$$

Hence C is quasi-p normal composition operator.

THEOREM-16: A composition operator C on $L^2(\mu)$ is quasi-p normal if and only if C^* is quasi-p normal.

THEOREM-17: A composition operator C on $L^2(\mu)$ is quasi-p normal if and only if $(C + C^*)$ commutes with M_{f_0}. Where M_{f_0} is the multiplication operator induced by $f_0 = (\frac{[illegible]}{[illegible]})$.

THEOREM-18: Let C be the quasi-p normal operator if and only if $(hoT).(foT) + h.E[h]oT^{-1}.E[f]oT^{-1} = h.(foT) + h^2 E[f]oT^{-1}$.

PROOF: Let C is quasi-p normal operator, then

$$(C + C^*)(C^*C) = (C^*C)(C + C^*)$$
$$(CC^*C + C^*C^*C) = (C^*CC + C^*CC^*)$$

Consider,

$$\begin{aligned}C(C^*C)f &= C.(h.f)\\ &= (hoT).(foT)\\ C^*(C^*C)f &= C^*.(h.f)\\ &= h.E[h]oT^{-1}.E[f]oT^{-1}\\ (C^*C)Cf &= C^*C(foT)\\ &= h.(foT)\\ (C^*C)C^*f &= (C^*C)(h.E[f]oT^{-1})\\ &= h^2.E[f]oT^{-1}\end{aligned}$$

Hence,C is a quasi-p normal if and only if, $(hoT).(foT) + h.E[h]oT^{-1}.E[f]oT^{-1} = h.(foT) + h^2 E[f]oT^{-1}$.

THEOREM-19: Let $C \in B(L^2(\lambda))$, then C^* is quasi-p normal operator if and only if $h.E[h].E[f]oT^{-1} + (ho\ T^2).E[f]oT = (hoT).E[h].E[f]oT^{-1} + (hoT).E[foT]$.

THEOREM-20: If C is quasi-n-normal and n-power normal operator, Then C is quasi-n-p normal composition operator.

THEOREM-21: Let C in $L^2(\mu)$ is quasi-n-p normal composition operator. Then C^* is quasi –n -p normal composition operator.

THEOREM-22: If C is quasi-n-p normal composition operator on $L^2(\mu)$. Then αC is quasi-n-p normal composition operator for every real number α.

THEOREM-23: A composition operator C on $L^2(\mu)$ is quasi-n-p normal if and only if $(C+C^*)$ commutes with $h.E[f]oT^{n-1}$.

THEOREM-24: Let C be quasi-n-p normal if and only if $(hoT).E[foT^n]+h.E[h]oT^{-1}.E[foT^{n-2}]=h.E[foT^n]$ $+h.E[h]oT^{-1}.E[f]oT^{n-2}$.

THEOREM-25: Let $C\in B(L^2(\lambda))$, Then C^* is the quasi-n-p normal operator if and only if $h.(h_n.E[f]oT^{-1})+(h_n oT^2).E[f]oT^{-(n-2)}=(hoT^{-(n-1)}).E[f]oT^{-(n-1)}$ $E[f]oT^{-n}+(h_n oT).E[foT^{-(n-2)}]$.

QUASI-P NORMAL AND QUASI–n-P NORMAL WEIGHTED COMPOSITION OPERATORS

Let W be the weighted composition operator on $L^2(\mu)$.

Let W^* be its adjoint which is given by $W^*f=h.E(u.f)\circ T^{-1}$ for $f\in L^2(\mu)$. For a positive integer n,$u_n=u.(u\circ T)2\ldots(u\circ T)^{(n-1)}$. For $f\in L^2(\mu)$, $W^n f=u_n.f\circ T^{-n}$ and $W^{*n}f=h_n.E(u_n.f)\circ T^{-n}$.

Proposition-26: For $u\geq o$;

(i) $W^*Wf=h.E[(u^2)]\circ T^{-1}.f$.

(ii) $WW^*f=u(h\circ T)E(u.f)$.

THEOREM-27: Let W be a weighted composition operators.Then W is quasi-p normal operator if and only if $u(hoT).E[u^2].(foT)$ $+h.E[u]oT^{-1}.E[h]oT^{-1}E[u^2]oT^{-2}.(foT^{-1})=h.E[u^2]oT^{-1}.$ $(u.foT)+h^2E[u^2]oT^{-1}.E[u.f]oT^{-1}$.

PROOF: Let W is a quasi-p normal operator, Then

$$(W+W^*)(W^*W)=(W^*W)(W+W^*)$$
$$(WW^*W+W^*W^*W)f=(W^*WW+W^*WW^*)f$$

Consider,

$$W(W^*W)f=W(h.E[u^2]oT^{-1}.f)=u(hoT).E[u^2].(foT)$$
$$W^*(W^*W)f=W^*(h.E[u^2]oT^{-1}.f)=h.E[u]oT^{-1}E[h]oT^{-1}E[u^2]oT^{-2}.(foT^{-1})$$
$$(W^*W)Wf=W^*W(u.foT)=h.E[u^2]oT^{-1}.(u.foT)$$
$$(W^*W)W^*f=W^*W(h.E[u.f]oT^{-1})=h^2E[u^2]oT^{-1}.E[u.f]oT^{-1}$$

Hence W is quasi-p normal operatorif and only if $u.(hoT).E[u^2].(foT)+h.E[u]oT^{-1}.E[h]oT^{-1}$ $E[u^2]oT^{-2}.(foT^{-1})=h.E[u^2]oT^{-1}.(u.foT)+$ $h^2E[u^2]oT^{-1}.E[u.f]oT^{-1}$.

THEOREM-28: Let W be a weighted composition operator. Then W^* is quasi-p normal operator if and only if $h.E[u^2]oT^{-1}.E[h].E[u.f]oT^{-1}+u.(uoT).(hoT^2).E[u.f]oT=$ $u.(hoT).E[u.h].E[u.f]oT^{-1}+u.(hoT).E[u^2].E[foT]$.

THEOREM-29: Let W be a weighted composition operators. Then W is quasi-n-p normal operator if and only if $u.(hoT).E[u.u_n].E[foT^n]+h.E[u.h]oT^{-1}.E[u.u_n]oT^{-2}$ $E[hoT^{n-2}]=h.E[u.u_n]oT^{-1}E[uoT^{n-1}]E[foT^n]$ $+h.E[u.u_n]oT^{-1}.E[hoT^{n-1}].E[u.f]oT^{n-2}$.

THEOREM-30: Let W be a weighted composition operators. Then W^* is quasi-n-p normal operator if and only if $h.E[u.u]oT^{-1}E[h_n].E[f.u_n]oT^{-n}+u(uoT)(h_n oT^2)$. $E[f.u_n]oT^{-(n-2)}=u.(h_n oT).E[u_n].E[h]oT^{-(n-1)}E[u.f]oT^{-n}$ $+u.(h_n oT).E[u.u_n]oT^{-(n-1)}E[foT]oT^{-(n-1)}$.

QUASI-P NORMAL AND QUASI–n-P NORMAL COMPOSITE MULTIPLICATION OPERATORS

A composite multiplication operator is a linear transformation acting on a set of complex valued measurable functions f of the form $M_{u,T}(f)=CM_u(f)=(uf)oT$. $=(uoT)(foT)$.Where u is a complex valued -measurable function. In case,$u=1$ almost everywhere $M_{u,T}$becomes a composition operator. The adjoint of $M_{u,T}$ is given by $M^*_{u,T}f=u.h.E[f]oT^{-1}$.

THEOREM-31: Let $M_{u,T}$ on $L^2(\lambda)$ be a composite multiplication operator. Then for each, $\lambda\geq 0$, $M_{u,T}$ is a quasi-p normal operator if and only if $(uoT)^2(hoT)E[u.f]oT+$ $u.h.E[u.h]oT^{-1}E[u.f]oT^{-1}=u.h.E[u].E[uoT].E[foT]$ $+u.h.E[u^2].E[h]E[f]oT^{-1}$.

THEOREM-32: Let $M_{u,T}$ on $L^2(\lambda)$ be a composite multiplication operator. Then for each, $\lambda\geq 0$, $M^*_{u,T}$ is quasi-p normal operator if and only if $u.h.E[u^2].E[h].E[f]oT^{-1}$ $+(uoT).(uoT^2).(uoT^2)(hoT^2)E[f]oT=(uoT)$ $((u.h).E[u.h].E[f]oT^{-1})+(uoT)^2(hoT)E[u.f]oT$.

THEOREM-33: Let $M_{u,T}$ on $L^2(\lambda)$ be a composite multiplication operator, Then for each $\lambda\geq 0$, $M_{u,T}$ is quasi-n-p normal operator if and only if $(uoT)^2.(hoT).E[(uoT).(uoT^2).(uoT^3)$ $\ldots\ldots(uoT^n)].(foT)+u.h.E[u]oT^{-1}E[(uoT).(uoT^2)(uoT^3)$ $\ldots\ldots(uoT^n)]oT^{-2}.(foT^{n-2})=u.hE[(uoT).(uoT^2).(uoT^3)$ $\ldots..(uoT^n)(uoT^{n+1})]oT^{-1}.(foT^n))+u.h.E[(uoT)(uoT^{n-1})$ $(uoT^n)(uoT^n)]oT^{-1}.E[h]oT^{n-1}.E[f]oT^{(n-2)}$.

THEOREM-34: Let $M_{u,T}$ on $L^2(\lambda)$ be a composite multiplication operator, Then for each $\lambda\geq 0$, $M^*_{u,T}$is quasi-n-p normal operator ifand only if $u.h.E[u.u.h].E[u.h]oT^{-1}.E[u.h]oT^{-2}$ $\ldots.E[u.h]oT^{-(n-1)}.E[f]oT^{-n}.+(uoT).(uoT^2).(uoT^2)(hoT^2)$ $((E[u.h].E[u.h]oT^{-1}\ldots.E[u.h]oT^{-(n-2)})oT.E[f]oT^{-(n-2)}$ $=(uoT^2).(hoT)E[u.h].E[u.h]oT^{-1}.E[u.h]oT^{-2}\ldots.E[u.h]$ $oT^{-(n-1)}.E[f]oT^{-n}+(uoT^2).(hoT).E[u.h].E[u.h]oT^{-1}.$ $E[u.h]oT^{-2}\ldots.E[u.h]oT^{-(n-2)}.E[f]oT^{-(n-2)}$.

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